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FUZZY NUMBERS AS AN ASSESSMENT TOOL IN THE APOS/ACE INSTRUCTIONAL TREATMENT FOR MATHEMATICS

Problem formulation. The APOS/ACE instructional treatment for learning and teaching mathematics was developed in the USA during the 1990's by a team of mathematicians and mathematics educators led by Ed Dubinsky ([1, 2], etc]. The implementation of the ACE cycle and its effectiveness in helping students making mental constructions and learn mathematics has been reported in several research studies of the Dubinsky's team (e.g. [17-19], etc) and by other researchers too (e.g. [5, 12], etc).

The **Fuzzy Sets Theory**, due to its nature of characterizing the ambiguous cases of the evolution of a phenomenon with multiple values, offers rich resources for the assessment purposes (e.g. see the books [4.10], etc). In an earlier paper [13], we have used the **Center of Gravity (COG) defuzzification technique** [7] for comparing the performances of two student groups concerning the comprehension of the real numbers in general and the irrational numbers in particular. The first group was taught the subject in the traditional way (control group), while the APOS/ACE instructional treatment was applied for the second group (experimental group).

In the present paper we describe an analogous experiment for comparing two university student groups' performance in learning mathematics, where we have used as assessment tool a combination of **Fuzzy Numbers (FNs)** and the COG technique. The rest of the paper is formulated as follows: In Section 2 we give a brief account of the APOS/ACE theory, while in Section 3 we expose the basic concepts and properties of FNs and in particular of the **Triangular Fuzzy Numbers (TFNs)** [15], that we are going to use for our purposes. In Section 4 we present our classroom experiment, while our last Section 5 is dedicated to our conclusion and to a brief discussion about the perspectives of future research on the subject.

The APOS/ACE Theory. APOS is a theory based on Piaget's principle that an individual learns (e.g. mathematics) by applying certain mental mechanisms to build specific mental structures and uses these structures to deal with problems connected to the corresponding situations [6]. Thus, according to the APOS analysis, an individual deals with a mathematical situation by using certain mental mechanisms to build cognitive structures that are applied to the situation. These mechanisms are called **interiorization** and **encapsulation** and the related structures are **Actions, Processes, Objects** and **Schemas**. The first letters of the last four words constitute the acronym APOS.

The theory postulates that a mathematical concept begins to be formed as one applies transformations on certain entities to obtain other entities. A transformation is first conceived as an action. For example, if an individual can think of a function only through an explicit expression and can do little more than substitute for the variable in the expression and manipulate it, he/she is considered to have an action understanding of functions.

As an individual repeats and reflects on an action, this action may be interiorized to a mental process. A process performs the same operation as the action, but wholly in the mind of the individual enabling her/him to imagine performing the transformation without having to execute each step explicitly. For example, an individual with a process understanding of a function thinks about it in terms of inputs, possibly unspecified, and transformations of those inputs to produce outputs.

When one becomes aware of a mental process as a totality and can construct transformations acting on this totality, then we say that the individual has encapsulated the process into a cognitive object. In case of functions encapsulation allows one to form sets of functions, to define operations on such sets, to equip them with a topology, etc.

Although a process is transformed into an object by encapsulation, this is often neither easy nor immediate. This happens because encapsulation entails a radical shift in the nature of one's conceptualization, since it signifies the ability to think of the same concept as a mathematical entity to which new, higher-level transformations can be applied. On the other hand, the mental process that led to a mental object through encapsulation remains still available and many mathematical situations require one to *de-encapsulate* an object back to the process that led to it. This cycle may be repeated one or more times. For example, in defining the sum f + g of two functions possessing a common domain, say A, it is necessary to reconsider again f and g at a process level and thinking of all x in A to obtain a new process associating to each x in A the sum f (x) + g(x). Then this new process must be encapsulated, in order to obtain the function f + g at an object level.

A mathematical topic often involves many actions, processes and objects that need to be organized into a coherent framework that enables the individual to decide which mental processes to use in dealing with a mathematical situation. Such a framework is called a schema. In the case of functions, for example, it is the schema structure that is used to see a function in a given mathematical or real-world situation. However, one must notice that there are not any rubrics in general to assess explicitly the level of understanding (by students) of mathematics corresponding to each cognitive level (structure) of the APOS theory. This is in fact a matter depending on the instructor's experience and intuition.

The APOS theory has important consequences for education. Simply put, it says that the teaching of mathematics should consist of helping students use the mental structures they already have to develop an understanding of as much mathematics as those available structures can handle. For students to move further, teaching should help them to build new, more powerful structures for handling more and more advanced mathematics.

Dubinsky and his collaborators realized that for each mental construction that comes out of an APOS analysis, one can find a computer task of writing a program or code, such that, if a student engages in that task, he (she) is fairly likely to build the mental construction that leads to learning the mathematics. In other words, performing the task is an experience that leads to one or more mental constructions. As a consequence of the above finding, the pedagogical approach based on APOS analysis, known as the **ACE teaching cycle**, is a repeated cycle of three components: **(A) activities on the computer**, **(C) classroom discussion** and **(E) exercises done outside the class**. The target of the activities on the computer is to help students in building the proper mental constructions for the better understanding and learning of the corresponding mathematical topic. The students discuss later in the classroom their experiences from the computer tasks performed in the laboratory, they repeat the same tasks without the help of computer and they reach, under their instructor's guidance and help, to the proper conclusions. Finally, the purpose of the exercises, which are given by the tutor as a home work, is to check and to embed better the new mathematical knowledge (for more details see [1, 2, 17], etc).

Fuzzy Numbers. The fuzzy sets theory was created in response of expressing mathematically real world situations in which definitions have not clear boundaries. For example, "the high mountains of a country", "the young people of a city", "the good players of a team", etc. The notion of a fuzzy set was introduced by Zadeh in 1965 [20] as follows:

1. Definition: A **fuzzy set** *A on the* universal set *U* (or a fuzzy subset of *U*) is a set of ordered pairs of the form $A = \{(x, m_A(x)): x \in U\}$, defined in terms of a **membership function** $m_A : U \to [0,1]$ that assigns to each element of *U* a real value from the interval [0,1].

The value $m_A(x)$ us called the **membership degree** of *x* in *A*. The greater is $m_A(x)$, the better *x* satisfies the characteristic property of *A*. The definition of the membership function is not unique depending on the user's subjective data, which is usually based on statistical or empirical observations. However, a necessary condition for a fuzzy set to give a reliable description of the corresponding real situation is that its membership function's definition is compatible to the laws of the common logic. Note that many authors, for reasons of simplicity, identify a fuzzy set with its membership function.

A crisp subset *A* of *U* can be consider as a fuzzy set in *U* with $m_A(x) = 1$, if $x \in A$ and $m_A(x) = 0$, if $x \notin A$. In this way most properties and operations of crisp sets can be extended to corresponding properties and operations of fuzzy sets. For general facts on fuzzy sets we refer to the book of Klir & Folger [4].

FNs play an important role in fuzzy mathematics, analogous to the role played by the ordinary numbers in classical mathematics. A FN is a special form of a fuzzy set on the set ***R*** of the real numbers defined as follows:

2. Definition: A **FN** is a fuzzy set *A* on the set ***R*** of real numbers with membership function $m_A: \mathbf{R} \to [0, 1]$, such that:

- *A* is **normal**, i.e. there exists *x* in ***R*** such that $m_A(x) = 1$,
- *A* is **convex**, i.e. all its *a*-cuts $A^a = \{x \in U: m_A(x) \geq a\}$, *a* in [0, 1], are closed real intervals, and
- The membership function $y = m_A(x)$ is a **piecewise continuous function**.

3. Counter example: Figure 1 represents the graph of a fuzzy set on ***R*** which is not convex. In fact, it is easy to observe that $A^{0.4} = [5, 8.5] \cup [11, 13]$, i.e. $A^{0.4}$ is not a closed interval.

Since the x-cuts of a FN, say *A*, are closed real intervals, we can write $A^x = [A_l^x, A_r^x]$ for each *x* in [0, 1], where A_l^x, A_r^x are real numbers depending on *x*. The following statement defines a *partial order* on the set of all FNs:

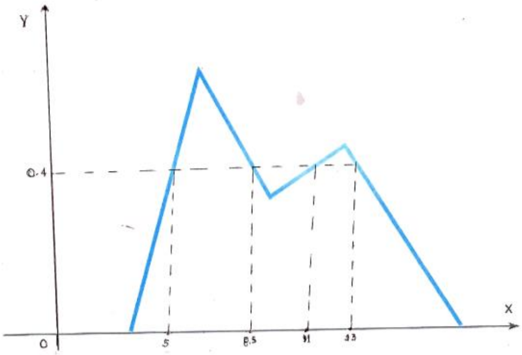

*Fig. 1 . Example of a non convex fuzzy set on **R***

4. Definition: Given the FNs *A* and *B* we write $A \leq B$ (or $\geq$) if, and only if, $A_l^x \leq B_l^x$ and $A_r^x \leq B_r^x$ (or $\geq$) for all x in [0, 1]. Two such FNs are called *comparable*, otherwise they are called *non comparable*.

5. Remark: One can define the four basic **arithmetic operations** on FNS in two, equivalent ways [3]: In practice, these two general methods of the fuzzy arithmetic, requiring laborious calculations, are rarely used in applications, where the utilization of simpler forms of FNs is preferred.

For general facts on FNs we refer to Chapter 3 of the book of Theodorou [8], which is written in Greek language, and also to the classical on the subject book of Kaufmann and Gupta [3].-

A TFN (*a, b, c*), with *a, b, c* in ***R*** is the simplest form of a FN. It actually means that *the value of b lies in the interval [a, c]*. The membership function of (*a, b, c*) is zero outside the interval [*a, c*], while its graph in [*a, c*] consists of two straight line segments forming a triangle with the OX axis (Figure 2). Therefore the analytical definition of a TFN is given as follows:

6. Definition: Let *a, b* and *c* be real numbers with *a* < *b* < *c*. Then the **TFN** (*a*, *b*, *c*) is a FN with membership function:

$$y = m(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b] \\ \dfrac{c-x}{c-b}, & x \in [b,c] \\ 0, & x < a \ or \ x > c \end{cases}$$

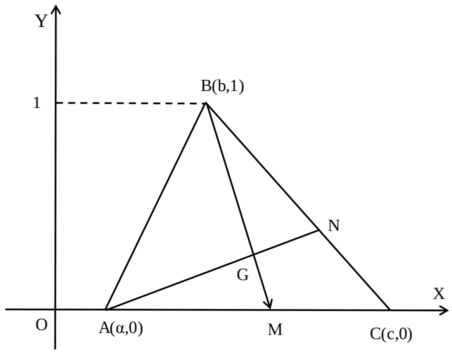

Fig. 2. Graph and COG of the TFN (a, b, c)

The following two Propositions refer to basic properties of TFNs that we are going to use later in this paper:

7. Proposition: The x-cuts A^x of a TFN $A = (a, b, c)$, x$\in$[0, 1], are calculated by the formula $A^x = [A_l^x, A_r^x] = [a + x(b - a), c - x(c - b)]$.

Proof: Since $A^x = \{y\in \boldsymbol{R}: m(y \geq x\}$, Definition 3.4 gives for the case of A_l^x that $\frac{y-a}{b-a}$ = x $\Leftrightarrow y = a + x(b - a)$. Similarly for the case of A_r^x we have that $\frac{c-y}{c-b}$ = x $\Leftrightarrow y = c - x(c - b)$.

8. Proposition: The coordinates (*X*, *Y*) of the COG of the graph of the TFN (*a*, *b*, *c)* are calculated by the formulas $X = \frac{a+b+c}{3}$, $Y = \frac{1}{3}$.

Proof: The graph of the TFN (*a*, *b*, *c)* is the triangle ABC of Figure 2, with A (*a*, 0), B(*b*, 1) and C (*c*, 0). Then, the COG, say G, of ABC is the intersection point of its medians AN and BM. The proof of the Proposition is easily obtained by calculating the equations of AN and BM and by solving the linear system of these two equations.-

9. Remark: The above proposition provides a **defuzzification** method of a TFN, i.e. its replacement by a crisp quantity (the coordinates of the COG of its graph).

10. Arithmetic Operations on TFNs: It can be shown [3] that the two general methods of defining arithmetic operations on FNs (see Remark 3.5) lead to the following simple rules for the **addition** and **subtraction** of TFNs:

Let $A = (a, b, c)$ and $B = (a_1, b_1, c_1)$ be two TFNs. Then

- The sum $A + B$ is the TFN $(a+a_1, b+b_1, c+c_1)$.
- The difference $A - B = A + (-B)$ is the TFN $(a-c_1, b-b_1, c-a_1)$, where $-B = (-c_1, -b_1, -a_1)$ is defined to be the **opposite** of *B.*

Obviously $A + (-A) = (a-c, 0, c-a) \neq O = (0, 0, 0)$, where the TFN *O* is defined by $O(x) = 1$, if $x = 0$ and $O(x) = 0$, if x$\neq$0. Note that, the **product** and the **quotient** of two TFNs, although they are FNs, they are not always TFNs, unless if a, b, c, a_1, b_1, c_1 are in $\boldsymbol{R}^+$ ([25], Section 3.2).

One can also define the following two **scalar operations**:

- k + *A*= (k+*a*, k+*b*, k+*c*), k$\in \boldsymbol{R}$
- k*A* = (k*a*, k*b*, k*c*), if k>0 and kA = (k*c*, k*b*, k*a*), if k<0.

We close this section by introducing the following definition, which will be used in Section 4 for assessing the student performance with the help of the TFNs:

11. Definition: Let A_i , i = 1, 2,…, *n* be TFNs, where *n* is a non negative integer, $n \geq 2$. Then we define the **mean value** of the A_i's to be the TFN

$$A= \frac{1}{n}(A_1 + A_2 + ... + A_n).$$

The Classroom Experiment. The following experiment was recently performed in the city of Patras with two student groups from the School of Technological Applications (prospective engineers) of the Graduate Technological Educational Institute (T. E. I.) of Western Greece, attending the course "Higher Mathematics I" (Calculus and Linear Algebra) of their first term of studies and having the same instructor. The students of both groups had more or less the same mathematical background from secondary education and were chosen in such a way that the grades, which they had obtained in the mathematics exam for entrance in higher education, were of about the same level. Also, since they were in their first term of studies, they had attended no previous mathematical courses at the T. E. I. of Western Greece.

The teaching procedure involved four didactic hours (45 minutes each) per week for each group. For the experimental group half of these hours were spent in a computer laboratory and the rest in the classroom according to the motive of the APOS/ACE instruction. On the contrary, for the control group the lectures were performed in the traditional way on the board, followed by a number of exercises and problems with the students participating for their solutions.

At the end of the term the students of both groups participated in the same final exam and the scores obtained, in a climax from 0 to 100, were the following:

Experimental Group (G_1): 100(2 times), 99(3), 98(5), 95(8), 94(7), 93(1), 92 (6), 90(5), 89(3), 88(7), 85(13), 82(6), 80(14), 79(8), 78(6), 76(3), 75(3), 74(3), 73(1), 72(5), 70(4), 68(2), 63(2), 60(3), 59(5), 58(1), 57(2), 56(3), 55(4), 54(2), 53(1), 52(2), 51(2), 50(8), 48(7), 45(8), 42(1), 40(3), 35(1).

Control Group (G_2)*:* 100(1), 99(2), 98(3), 97(4), 95(9), 92(4), 91(2), 90(3), 88(6), 85(26), 82(18), 80(29), 78(11), 75(32), 70(17), 64(12), 60(16), 58(19), 56(3), 55(6), 50(17), 45(9), 40(6).

The student performance was characterized by the fuzzy linguistic labels (grades) A, B, C, D and F corresponding to the above scores as follows: A (85-100) = excellent, B (84-75) = very good, C (74-60) = good, D (59-50) = fair and F (<50) = unsatisfactory.

Next, we assigned to each linguistic label (grade) a TFN (denoted, for simplicity, by the same letter) as follows: A= (85, 92.5, 100), B = (75, 79.5, 84), C = (60, 67, 74), D= (50, 54.5, 59) and F = (0, 24.5, 49). The middle entry of each of the above TFNs is equal to the mean value of the student scores attached to the corresponding linguist label (grade). In this way a TFN corresponds to each student assessing his (her) individual performance. The representation of the linguistic labels A, B, C, D and F by TFNs has the advantage of determining numerically the scores corresponding to each label. In fact, the scores assigned to the above labels in the present application are not standard, since they may differ from case to case in practice. For example, in a more rigorous assessment, one could take A(90-100), B (80-89), C(70-79), D (60-69), F(<60), etc.

We now form the following Table 1 depicting the students' performance in terms of the TFNs defined above:

Table 1

Students' performance in terms of the TFNs

TFN	G_1	G_2
A	60	60
B	40	90
C	20	45
D	30	45
F	20	15
Total	170	255

We observe that in Table 1 we have 170 TFNs representing the progress of the students of G_1 and 255 TFNs representing the progress of the students of G_2. Therefore, it is logical to accept that the overall performance of each group is given by the corresponding mean value of the above TFNs (Definition 11). For simplifying our notation, let us denote the above mean values by the letter of the corresponding group. Then, making the necessary straightforward calculations, one finds that:

$G_1 = \frac{1}{170}$. (60A+40B+20C+30D+20F) $\approx$ (63.53, 71.74, 83.47) and

$G_2 = \frac{1}{255}$. (60A+90B+45C+45D+15F) $\approx$ (65.88, 72.63, 79.53).

Observing the left entries (63.53 and 65.88 respectively) and the right entries (83.47 and 79.53 respectively) of the TFNs G_1 and G_2 one concludes that the overall performance of the two groups could be characterized from good (C) to very good (B). It is also of worth to clarify that the middle entries of G_1 and G_2 (71.74 and 72.63 respectively) give a **rough approximation** only of each group's overall performance. In fact, since the middle entries of the TFNs A, B, C, D and F were chosen to be equal to the means of the scores assigned to the corresponding linguistic grades, the middle entries of the TFNS G_1 and G_2 are simply equal to the mean values of these means and therefore they **do not measure the mean performances** of the two groups. In fact, calculating the means of the student scores in the classical way one finds the values 72.44 and 72.04 for G_1 and G_2 respectively, showing a slight superiority of the experimental group.

Next, applying Proposition 7 one finds that the x-cuts of the two TFNs are G_1^x = [63.53+8.21x, 83.47-11.73x] and G_2^x = [65.88+6.75x, 79.53-6.9x] respectively. But $63.53+8.21x \le 65.88+6.75x \Leftrightarrow 1.46x \le 2.35 \Leftrightarrow x \le 1.61$, which is true, since x is in [0, 1]. On the contrary, $83.47-11.73x \le 79.53-6.9x \Leftrightarrow 3.94 \le 4.83x \Leftrightarrow 0.82 \le x$, which is not true for all the values of x. Therefore, according to Definition 3.4, the TFNs G_1 and G_2 are not comparable, which means that in this stage one **can not decide which of the two groups demonstrated the better performance.**

A good way to overcome this difficulty is to defuzzify the TFNs G_1 and G_2. In fact, by Proposition 8, the COGs of the graphs of the TFNs G_1 and G_2 have x-coordinates equal to $X = \frac{63.53+71.74+83.47}{3} \approx 72.91$ and $X' = \frac{65.88+72.63+79.53}{3} \approx 72.68$ respectively.

Observe now that these GOGs lie in a rectangle with sides of length 100 units on the X-axis (student scores from 0 to 100) and one unit on the Y-axis (normal fuzzy sets). Therefore, **the nearer the x-coordinate of the COG to 100, the better the corresponding group's performance**. Thus, since $X > X'$, the experimental group G_1 demonstrates a (slightly) better overall performance than the control group G_2.

Conclusion. In the present paper we used the TFNs for comparing the difference of university students' performance when learning mathematics with the APOS/ACE instructional treatment (experimental group), as well as with the traditional way on the board (control group). In our case, in contrast to earlier experimental researches of the Dubinsky's team and of other researchers, no significant difference was found for the performance of the two groups. In fact, the slight superiority of the experimental group is not enough to obtain secure conclusions and therefore a further investigation seems to be needed on the subject.

Since two TFNs are not always comparable, the use of them as assessment tools was combined here with the COG defuzzification technique. The creditability of this new fuzzy assessment approach was validated by comparing its outcomes in our application with the corresponding mean values of the students' performance, i.e. the most standard assessment method of the traditional, bi-valued logic.

The combination of the TFNs with the COG technique seems to have the potential of a *general assessment method* that could be used in a great variety of other machine (e.g. for decision-making [14], case-based reasoning [9], computational thinking [11] etc) and human (e.g. players' assessment [16], etc) activities. This is indeed the main target of our future research on the subject.

References

1. Asiala, M., et al., A framework for research and curriculum development in undergraduate mathematics education, Research in Collegiate Mathematics Education II, CBMS Issues in Mathematics Education, 6, 1-32, 1996.
2. Dubinsky, E. & McDonald, M. A., APOS: A constructivist theory of learning in undergraduate mathematics education research. In: D. Holton et al. (Eds.), The Teaching and learning of Mathematics at University Level: An ICMI Study, 273-280, Kluwer Academic Publishers, Dordrecht, Netherlands, 2001.
3. Kaufmann, A. & Gupta, M., Introduction to Fuzzy Arithmetic, Van Nostrand Reinhold Company, New York, 1991.
4. Klir, G. J. & Folger, T. A., Fuzzy Sets, Uncertainty and Information, Prentice-Hall, London, 1988.
5. Maharaj, A., An APOS analysis of natural science students' understanding of derivatives, South African Journal of Education, 33(1), 19 pages, 2013.
6. Piaget, J., *Genetic Epistemology*, Columbia University Press, New York and London, 1970.
7. Subbotin, I. Ya. Badkoobehi, H., Bilotckii, N. N., Application of fuzzy logic to learning assessment. Didactics of Mathematics: Problems and Investigations, 22, 38-41, 2004.
8. Theodorou, J., Introduction to Fuzzy Logic, Tzolas Publications, Thessaloniki, Greece, 2010 (in Greek language).
9. Voskoglou, M. Gr., Fuzzy Sets in Case-Based Reasoning, Fuzzy Systems and Knowledge Discovery, Vol. 6, 252-256, IEEE Computer Society, 2009.
10. Voskoglou, M. Gr., Stochastic and fuzzy models in Mathematics Education, Artificial Intelligence and Management, Lambert Academic Publishing, Saarbrucken, Germany, 2011.
11. Voskoglou, M. Gr., Problem solving, fuzzy logic and computational thinking, Egyptian Computer Science Journal, 37(1), 131-145, 2013.
12. Voskoglou, M. Gr., An application of the APOS/ACE approach in teaching the irrational numbers, Journal of Mathematical Sciences and Mathematics Education, 8(1), 30-47, 2013.
13. Voskoglou, M. Gr., Fuzzy Logic in the APOS/ACE Instructional Treatment of Mathematics, American Journal of Educational Research, 3(3), 330-339, 2015.
14. Voskoglou, M. Gr., Using the Triangular Fuzzy Numbers for the Verification of a Chosen Decision, Academic Journal of Applied Mathematical Sciences, 1(1), 1-8, 2015.
15. Voskoglou, M. Gr., Use of the Triangular Fuzzy Numbers for Student Assessment, American Journal of Applied Mathematics and Statistics, 3(4), 146-150, 2015
16. Voskoglou, M. Gr. & Subbotin, I. Ya., Fuzzy Numbers: A Tool for the Assessment of Human Skills, International Journal of Applications of Fuzzy Sets and Artificial Intelligence, 6, 17-32, 2016
17. Weller, K. et al., Students performance and attitudes in courses based on APOS theory and the ACE teaching cycle. In: A. Selden et al. (Eds.), Research in collegiate mathematics education V (pp. 97-181), Providence, RI: American Mathematical Society, 2003.
18. Weller, K. , Arnon, I & Dubinski, E. , Pre-service Teachers' Understanding of the Relation Between a Fraction or Integer and Its Decimal Expansion, Canadian Journal of Science, Mathematics and Technology Education, 9(1), 5-28, 2009.

19. Weller, K., Arnon, I & Dubinski, E., Pre-service Teachers' Understanding of the Relation Between a Fraction or Integer and Its Decimal Expansion: Strength and Stability of Belief, Canadian Journal of Science, Mathematics and Technology Education, 11(2), 129-159, 2011.
20. Zadeh, L. A., Fuzzy Sets, *Information and Control*, 8, 338-353, 1965.

Анотація. Воскоглой М. Гр. Нечіткі числа як інструмент оцінки APOS/ACE методів навчання математики.

У статті використовується комбінація методів трикутних нечітких чисел (TFNs) та центру тяжіння (COG) як техніки дефазифікації для оцінки знань і навичок студентів універистету у процесі навчання математики у рамках APOS/ ACE.

Ключові слова: *APOS/ACE методи навчання математики, трикутні нечіткі числа (TFNs), центр тяжіння (COG), техніка дефазифікації.*

Аннотация. Воскоглой М. Гр. Нечеткие числа в качестве инструмента оценки APOS/ACE методов обучения математики.

В статье используется комбинация методов треугольных нечетких чисел (TFNs) и центра тяжести (COG) как техники дефаззификации для оценки навыков и знаний студентов университета в процессе обучения математике в рамках APOS/ ACE.

Ключевые слова: *APOS/ACE методы обучения математики, треугольные нечеткие числа (TFNs), центр тяжести (COG) техника дефаззификации.*

Abstract. Voskoglou M. Gr. Fuzzy Numbers as an Assessment Tool in the APOS/ACE Instructional Treatment of Mathematics

In the article a combination is used of of the Triangular Fuzzy Numbers (TFNs) and the Center of Gravity (COG) defuzzification technique to assess university student skills for learning mathematics with the APOS/ACE instructional treatment.

Key words: *APOS/ACE instructional treatment of mathematics, triangular fuzzy numbers (TFNs), center of gravity (COG) defuzzification technique*.